A Smart Polynomial Graphing Applet

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Abstract

This applet graphs polynomials of degree four or less, and it automatically chooses a viewing window that shows the significant features of the graph. There are many other polynomial graphing applets available, but we are not aware of any other applet that chooses the viewing window. We provide several routine exercises that allow students to use the applet to study some fundamental concepts from algebra and calculus. We also provide more advanced exercises that give the student a feel for how the applet chooses the viewing window.

Intended Audience

This material is appropriate for undergraduate students in a precalculus or calculus course. The more advanced exercises are appropriate for students in Calculus II or higher.

System Requirements

The applet was compiled using Sun Microsystems Java 2 SDK Version 1.4.1_02™ and requires that your browser be capable of running Java 2 applets. If your browser does not support Java 2 Version 1.4 applets, then you must download and install the latest release of the Sun Java Plug-In from Sun's web site. We have tested this applet using Internet Explorer Version 6 (on Windows NT), Netscape Version 7 (on Windows NT and Redhat Linux), and Mozilla Version 1.4 (on Windows NT and Redhat Linux). If you have difficulties running the applet, you should upgrade your browser software to the latest version, and install the latest Sun Java plug-in. These pages are compliant with HTML Version 4 and CSS Version 2. If you have trouble viewing them, make certain your browser supports HTML4 and CSS2.

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0. Note to Teachers

We hope students will use the applet in two different ways. First, students in precalculus and calculus can use the applet to quickly and easily verify their calculations in routine exercises such as finding intercepts, counting the real and non-real zeros, completing the square, working with the rational roots theorem and Descrartes' rule of signs, and finding local extrema and inflection points.

Our second aim, which is the driving force behind this project, is to have students explore how the applet works. In the advanced exercises, students will use basic facts from algebra and calculus to determine an appropriate viewing window for some example polynomials. Students can then compare their viewing window with the one chosen by the applet.

1. Introduction

Given a polynomial with real coefficients, can we use the coefficients to determine an appropriate viewing window for the graph? We address this question for polynomials of degree 4 or less using the following rules:

- The viewing window should include the significant features of the graph, such as intercepts, extrema, etc.
- The origin should be included in the window.
- Only basic algebra and calculus should be used. The quadratic formula is allowed, but Cardan's formulas, Ferrari's formulas, and sophisticated numerical techniques are not allowed.

Given the coefficients of a polynomial, the applet determines a suitable viewing window and draws the graph in that window. There is an abundance of applets that graph polynomials, but we are not aware of any other applet which automatically chooses the viewing window. With this applet, students can explore several important features of polynomial functions.

2. Precalculus and Calculus Exercises

The applet graphs polynomials of the form $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, where each a_i is real. In these exercises students can use the applet to verify routine calculations. In the advanced exercises, students can explore how the applet chooses the viewing window.

Exercise 1 - Quadratic Functions

Without the help of a calculator or computer, find the zeros, find the coordinates of the vertex, and sketch the graph. Use the applet to verify your work.

a. $f(x) = 2x^2 - 80x + 700$

b. $f(x) = 3x^2 + 3x + 11$

c. $f(x) = -.5x^2 + 3x + 30$

Exercise 2 - Rational Zeros Theorem

List all possible rational zeros. Then use the applet to decide which numbers on your list are the rational zeros. If possible, find all remaining real and non-real zeros.

a. $f(x) = 6x^4 - 5x^3 - 5x - 6$

b. $f(x) = x^3 - x^2 - 30x + 72$

c. $f(x) = x^3 - 35x^2 - 34x - 72$

Exercise 3 - Descartes' Rule of Signs

List the possible number of positive and negative real roots. Then use the applet to make sure your answers are consistent with the graph.

a. $f(x) = x^4 + x^3 - x^2 - x + 1$

b. $f(x) = x^4 + x^3 - x^2 - x + .5$

c. $f(x) = x^4 + x^3 - x^2 - x$

Exercise 4 - Features of the Graph of a Function

For each of the following, find the zeros, critical points, and inflection points. Identify intervals where f is positive/negative, increasing/decreasing, and concave up/down. Then use the applet to verify your work.

a. $f(x) = 2x^3 + 3x^2 - 12x$

b. $f(x) = x^4 - 9x^3 + 20x^2$

c. $f(x) = -2x^4 + x$

Exercise 5 - Limitations of the Applet

We must always be aware of the limitations of any technology we use. The following questions help to reveal some of the short-comings of this applet.

a. *Zeros Away From Origin* Let $f(x) = (x - 20)(x - 21)(x - 22)(x - 23)$. What do you expect the graph of this function to look like? Sketch the graph by hand. If we expand $f(x)$, we obtain $f(x) = x^4 - 86x^3 + 2771x^2 - 39646x + 212520$. Now use the applet to graph the function. Are there some important details missing in the applet's graph?

b. *Large Constant term* Compare the graphs of $f(x) = x^3 - 3x^2 + 2x$ and $f(x) = x^3 - 3x^2 + 2x^2 + 2000$. How does a large constant term affect the accuracy of the applet's graph?

c. *Varied gaps between zeros.* Let $f(x) = x^4 - 199x^3 + 9900x^2$. Find the zeros of f by hand. Then use the applet to graph the function. Are all three zeros visible in the applet's graph?

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3. Advanced Exercises

This applet uses the coefficients of a polynomial to find a suitable viewing window for its graph. We used the following rules while designing the applet:

- The viewing window should include the significant features of the graph, such as intercepts, extrema, and inflection points.
- The origin should be included in the window.
- Only basic algebra and calculus should be used. The quadratic formula is allowed, but Cardan's formulas, Ferrari's formulas, and sophisticated numerical techniques are not allowed.

The following exercises will help you get a feel for how the applet chooses the viewing window. Work these problems without the help of the applet. Then compare your values with the ones chosen by the applet.

Exercise 1 - Including the Zeros

a. Find real numbers *xzeromin* and *xzeromax* such that the zeros of $f(x) = x^4 - 5x^3 - 18x^2 + 32x$ all lie between *xzeromin* and *xzeromax*.

Hint: Since

$$\lim_{x\to\infty}\frac{5x^3+18x^2-32x}{x^4}=0$$

there is an $N > 0$ such that for $x > N$, we have $5x^3 + 18x^2 - 32x < x^4$. (hence $f(x) > 0$). Here is one way to find such an N. First, note that for $x > 0$, $5x^3 + 18x^2 - 32x < 5x^3 + 18x^2$. Now find N_1 and N_2 such that $5x^3 < .5x^4$ for $x > N_1$ and $18x^2 < .5x^4$ for $x > N_2$, and let N (= *xzeromax*) be the larger of N_1 and N_2. Then for $x > N$, we have $5x^3 + 18x^2 < .5x^4 + .5x^3 = x^4$. Use a similar approach to find *xzeromin*, keeping in mind that $x < 0$ in this case.

b. Use the techniques suggested in the previous problem (or devise better techniques!) to find *xzeromin* and *xzeromax* for the following polynomials.

   i. $f(x) = -x^4 + 3x^3 + 7x^2 - 11x$

   ii. $f(x) = 5x^3 + 9x^2 - 20x + 10$

   iii. $f(x) = x^4 + 2x^3 + 3x^2 - 2x + 1$

   iv. $f(x) = x^4 + 2x^3 - 3x^2 - 2x + 1$

c. For each of the functions given in the previous two exercises, use the derivative f' and the second derivative f'' to find real numbers *xcritmin*, *xcritmax*, *xinfmin*, and *xinfmax* so that the critical points of f lie between *xcritmin* and *xcritmax* and the inflection points of f lie between *xinfmin* and *xinfmax*. Use your answers to find numbers *xmin* and *xmax* so that all the significant features of f will occur between *xmin* and *xmax*.

Exercise 2 - Finding *y* Values

a. If f is a cubic polynomial, we can use the quadratic formula to find the critical points (if there are any). Then we can evaluate f at *xmin*, *xmax*, and the critical points to find appropriate values for *ymin* and *ymax*. Find *ymin* and *ymax* for $f(x) = 5x^3 + 9x^2 - 20x^2 + 10$.

b. If f is a quartic polynomial, we cannot easily find the critical points, but the second derivative, which is quadratic, will indicate if there are any inflection points and give us a good deal of information regarding the shape of the graph. The only point on the graph that we can always find is the zero of the third derivative, which is in some sense the "straightest" point on the graph (we call it *SpecialPoint*).

   i. Case quartic with no inflection points. Find *ymin* and *ymax* for $f(x) = x^4 + 2x^3 + 3x^2 - 2x + 1$ **Hint:** Build the tangent line at *SpecialPoint*, and find the points on this line corresponding to *xmin* and *xmax*.

   ii. Case quartic with two inflection points. Find *ymin* and *ymax* for $f(x) = x^4 + 2x^3 - 3x^2 - 2x + 1$ **Hint:** Build the tangent line at the two inflection points, and use these lines to guide your choice of *ymin* and *ymax*.

Exercise 3 - Finding the Viewing Window I

Find a suitable viewing window for the following polynomials:

a. $f(x) = 57x - 93$

b. $f(x) = x^2 - 20x + 70$

c. $f(x) = x^3 - 30x^2 + 200x - 500$

d. $f(x) = -50x^3 + 8x^2$

e. $f(x) = 100x^4 - 20x^3 + x$

f. $f(x) = x^4 + 20x^2 + 400x - 10000$

Exercise 4 - Finding the Viewing Window II

If f has no inflection points and its shape at *SpecialPoint* is zero, the tangent line at *SpecialPoint* is of no use to us. Develop techniques to find a viewing window for the following:

a. $f(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$

b. $f(x) = x^4 + 4x^3 + 6x^2 + 4x + 100$

c. $f(x) = 2x^4 + 10x^2$

d. $f(x) = .1x^4 + 3$

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4. How to Use the Applet

We hope you find the applet is self-explanatory and easy to use, but just in case you need some help, in this section we provide instructions on how to use the applet.

4.1 Entering Values for Coefficients

When the applet window appears, you may enter the coefficients for your polynomial in the text fields at the bottom of the applet window. If a text field is left blank, the corresponding coefficient will be set to zero. After entering the coefficients, to plot the polynomial in the viewing window, either press the *Enter* key within one of the text fields, or click on the *Plot f(x)* button in the lower left corner of the applet window. The applet will then determine the appropriate viewing window, and will plot the specified polynomial -- see Figure 1.

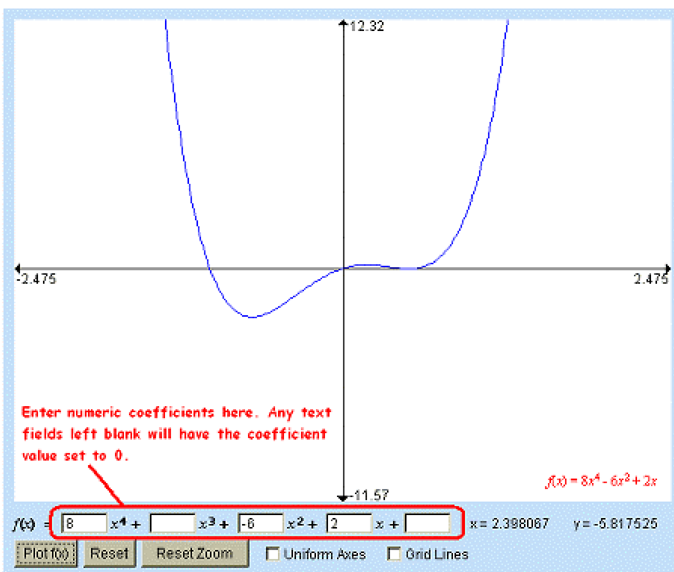


Figure 1. Entering Coefficient Values

4.2 The Uniform Axes Checkbox

By default, the viewing window for the specified polynomial may not be proportional in the X- and Y- directions. The viewing window coordinates are selected based on the characteristics of the specified polynomial such that the viewing window will show the relevant features of the polynomial. For example, notice in Figure 1 above that the displayed X-axis ranges from -2.475 to +2.475 (a spread of 4.95) whereas the displayed Y-axis ranges from -11.57 to +12.32 (a spread of 23.89). The *Uniform Axes* checkbox at the bottom of the applet window can be selected to change the viewing window so it is proportional in the X- and Y- directions. When this checkbox is selected, the viewing window coordinates will be proportional, and when it is not selected, the viewing window coordinates are non-proportional. Figure 2 shows how the polynomial $f(x) = 8x^4 - 6x^2 + 2x$ would be plotted when *Uniform Axes* is selected.

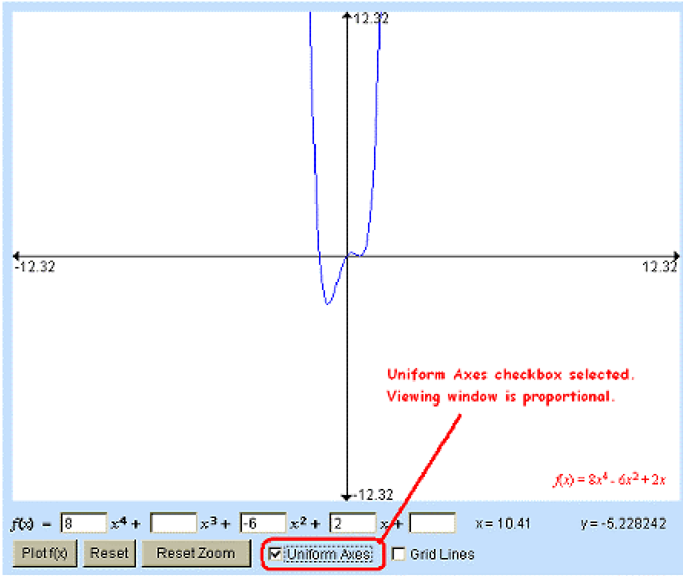


Figure 2. Selecting a Proportional Viewing Window with Uniform Axes

4.3 The Grid Lines Checkbox

Selecting the *Grid Lines* checkbox at the bottom of the applet window will plot grid lines in the polynomial viewing window -- see Figure 3.

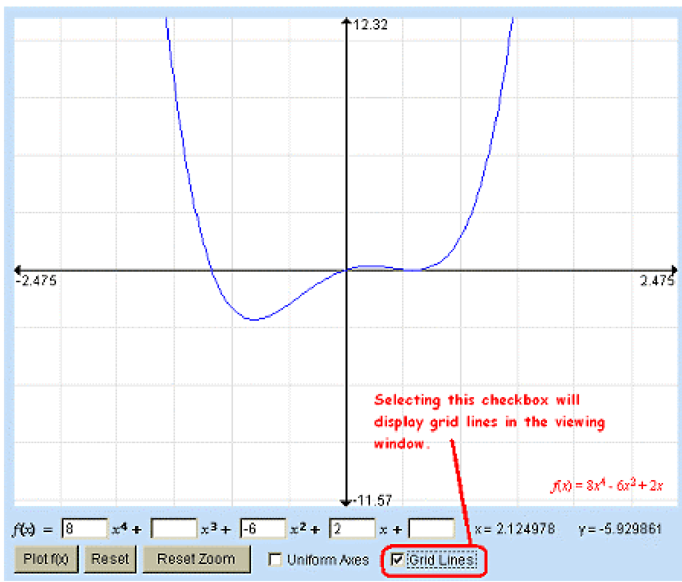


Figure 3. Plotting Grid Lines

4.4 Identifying (X,Y) Coordinates

As the mouse pointer is moved around in the viewing window, the (X,Y) coordinates of the mouse pointer will be displayed in the lower right part of the applet window -- see Figure 4.

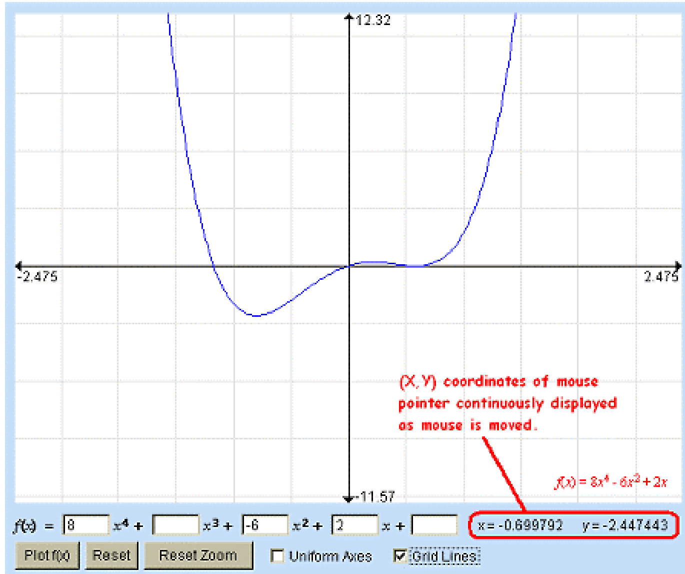


Figure 4. Identifying (X,Y) Coordinates

4.5 Zooming In

Even though the applet selects viewing window coordinates which cause the relevant features of the polynomial to be displayed, there may be times when you would like to "zoom in" a bit closer on a specific part of the graph. Zooming in is initiated by clicking the first mouse button (the left mouse button on a right-handed mouse) once and holding down. As the mouse button is held down and the mouse is moved, a bounding rectangle will be displayed. This bounding rectangle specifies the new viewing window which will be selected when the mouse button is released -- see Figures 5 and 6. *Note that if Uniform Axes is selected when a zoom operation is performed, then after the zoom operation completes, Uniform Axes will no longer be selected. This is so because the bounding rectangle most likely will not be square.*

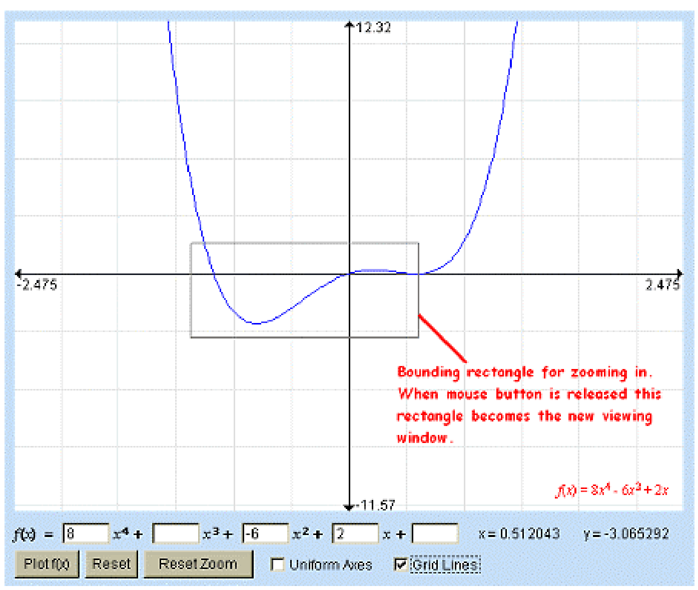


Figure 5. Selecting a Bounding Rectangle for Zooming In

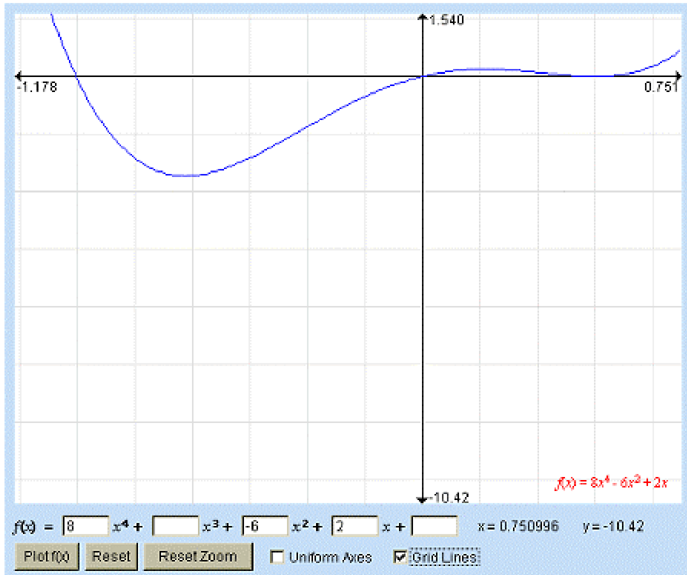


Figure 6. Zooming In (Continued from Figure 5)

4.6 Limits on Zooming and Resetting the Zoom Level

There is a limit as to how much you can zoom in on a particular feature of the graph. For example, if we zoom in on the part around the positive X-axis, we obtain a new viewing window as shown in Figures 7 and 8. If you continue to zoom in again and again on successively smaller regions, however, eventually the applet will not allow any more zoom operations. At this point, the applet will display a dialog window informing you that you have reached the maximum zoom limit, see Figure 9. At this point the applet will redraw the viewing window at the last selected zoom level. You can reset the viewing window coordinates back to their original values by clicking on the *Reset Zoom* button.

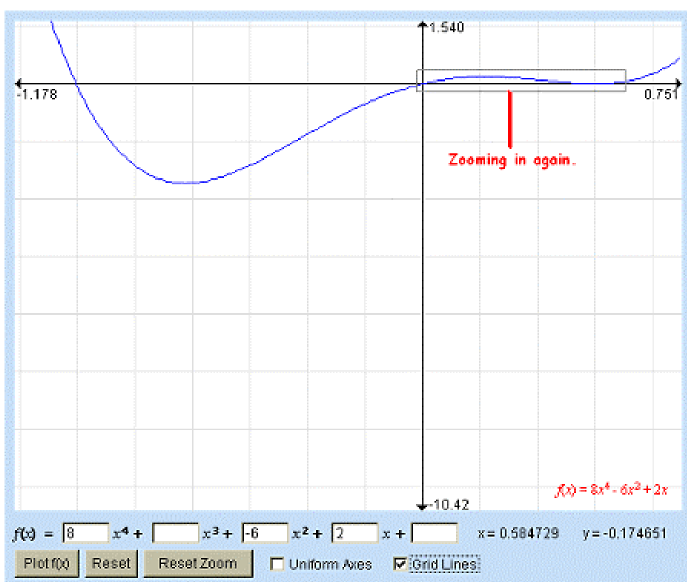
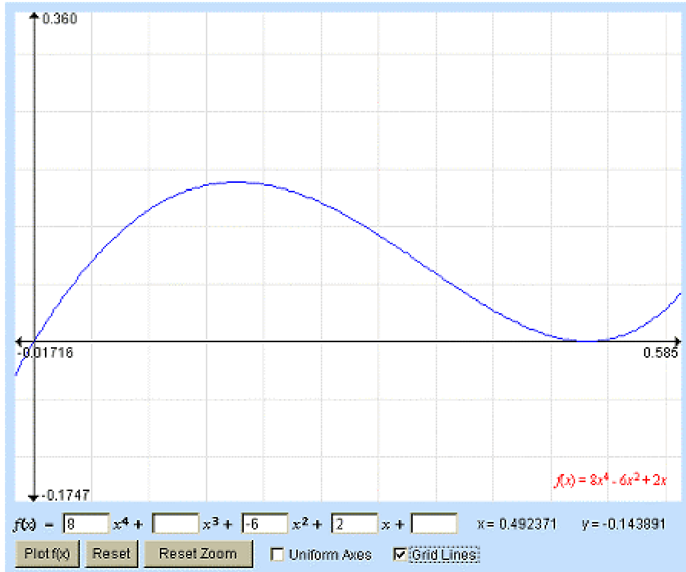


Figure 7. Zooming In Again



Figures 8. Zooming In Again (Continued from Figure 7)

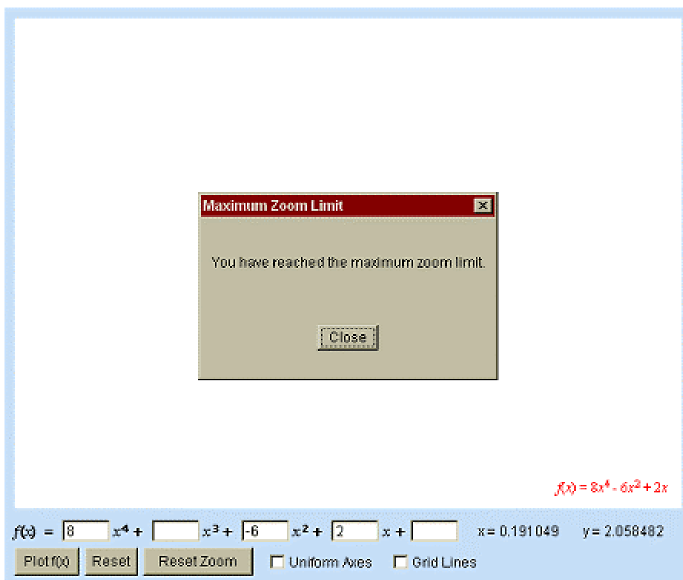


Figure 9. Maximum Zoom Limit Has Been Reached

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6. Known Bugs and Limitations

As bugs or limitations are discovered we will document them here. When time permits, serious bugs or limitations will be corrected. Minor bugs and limitations may never be corrected.

6.1 Problem with Zooming in with Grid Lines Selected (Minor Bug)

If the *Grid Lines* checkbox is selected and multiple zoom operations on smaller and smaller regions are performed, then at some point it begins to take a long time to plot the grid lines. The applet will appear to be locked up, but it is really trying to plot the grid lines. This only appears to happen at extreme zoom-in levels. We consider this a minor bug, because zooming in on such a small region is not a very useful operation, and so will happen very seldom in practice. This does not occur when *Grid Lines* is cleared.

6.2 Repaint Problem with Some Browsers (Minor Limitation)

If you are using Internet Explorer Version 6 or Mozilla version 1.4 and you resize the browser window when the applet is running, the applet window may not be repainted correctly. This does not happen in Netscape Version 7, so it appears to be a problem with Internet

Explorer and Mozilla - perhaps those browsers do not send the applet a repaint() message when the window is resized. In any case, we do not know any way to change the applet code to work around the problem. If you do, please email us at the email addresses listed at the bottom of this page. Of course, invoking the *Refresh* or *Reload Page* command in your browser window *does* send the applet a repaint() command and the window will be redrawn correctly.

6.3 Problem Using Uniform Axes Feature (Minor Bug)

When the *Uniform Axes* checkbox is selected, the features of the plotted graph may not always be evident because of the way the *x*- and *y*-ranges are selected. This uniform axes feature was implemented primarily for plotting linear polynomials. When plotting lines with nonuniform axes the *apparent* slope of the plotted line is different than the *actual* slope and we were concerned this disparity might be confusing for students.

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7. Source Code

The applet was written in Java using the Sun Java 2 SDK Version 1.4.1_02™. The source code is available here in zip format and as a jar file. We are releasing this applet under the GNU General Public License (GPL). Please support high-quality open source software.

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8. References and Links

Below is a list of other Java applets we found that also plot polynomials. As far as we know, there are no applets which graph a polynomial by automatically selecting the viewing window in the manner ours does.

- http://www.univie.ac.at/future.media/moe/fplotter/fplotter.html
- http://www.langara.bc.ca/mathstats/resource/GraphExplorer
- http://www.math.umn.edu/~garrett/qy/Quintic.html
- http://conceptengineering.com/dan/Polynomial3.html
- http://conceptengineering.com/dan/Polynomial4.html
- http://library.thinkquest.org/C004647/calculator/polynomial/polynomial.html
- http://xanadu.math.utah.edu/java/CubicGraph.html
- http://www.math.utah.edu/~carlson/teaching/java/calculus/CubicGraph.html
- http://www.ugrad.math.ubc.ca/coursedoc/math102/java/m102/demos/polygfx/poly.html
- http://www.rpi.edu/dept/chem-eng/Biotech-Environ/Canada/polynom.html
- http://id.mind.net/~zona/ezGraph/ezGraph.html
- http://mss.math.vanderbilt.edu/cgi-bin/MSSAgent/~pscrooke/MSS/solvepoly.def

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